Max Marks : 250 **Duration : 3 Hours**

MATHEMATICS

Q.1 Find the units digit of 777^{777}.

Q.2 In the standard expansion of $(\sum_{i=0}^{28}(-1)^i a_i)^2$, the number of terms appearing with negative sign is _____

Q.3 If a line, parallel to, but not identical with, x- axis cuts the graph of the curve $y = \frac{x-1}{(x-2)(x-3)}$ at $x = a, x = b$, then evaluate $(a-b)(b-1)$

Q.4 Does there exist a right angled triangle with integral sides such that the hypotenuse measures 2000 units of length?

Q.5 ABCD is a trapezium with AB and CD as parallel sides. The diagonals intersect at O. The area of the triangle ABO is p and that of the triangle CDO is q.
Prove that the area of the trapezium is $(\sqrt{p}+\sqrt{q})^2$

Q.6 Numbers 1, 2, 3 ……, 2009 are written in the natural order. Numbers in odd places are stricken off to obtain a new sequence. Numbers in odd places are stricken off from this sequence to obtain another sequence and so on, until only one term a is left. Then find a

Q.7 Given a set of 'n' rays in a plane, we mean by 'a reversal' the operation of reversing precisely one ray and obtaining a new set of 'n' rays. Starting from 2009 rays and performing one million reversals, is it possible to reverse all the rays?

Q.8 We know that we can triangulate any convex polygonal region. Can we 'parallelogramulate' a convex region bounded by a 2009-gon?

Q.9 Solve in positive integers the cubic $(x^3+2^3)-(x+1)^2=2009.$

Q.10 The product of two of the roots of $x^4-11x^3+kx^2+269x-2001$ is -69. Find k.

PHYSICS

Q.1 Two small stones are tied at the ends of a rope of length 5 m. Imagine you are holding them, one in each hand, with the rope hanging loosely on the top floor balcony of a tall building. You release one stone at a certain moment and the other 0.5 s later along the same vertical line from the same position. The rope will be taut after a time of _____. Assume that as the rope gets taut, the velocities of the stones becomes equal to the average of the velocities of the stones just before the stone gets taut. The velocities of the stones 2 s later would be _____.

Q.2 A cylindrical wire of radius R is made of two different materials. The inner portion of radius R/2 is made of a material of resistivity ρ and the outer annular portion of thickness R/2 is made of material of resistivity 2ρ. Resistance per unit length of this wire would be_____.

Q.3 A boat moves down stream a certain distance in a river. If the river were not flowing the time taken would have been 2 hrs but with the river flowing the time taken is 1.5 hrs. The time taken to travel upstream the same distance would be _____.

Q.4 A sphere consists of inner portion of radius r made of a material of density 8 gm/cm^3, and an outer layer of thickness t made of a material of density 4 gm/cm^3. This sphere floats in a liquid of density 10 gm/cm^3 with half its volume submerged. The ratio r/t is equal to _____.

Q. 5 A radio active substance of half life period 2 hrs is present in a reactor. At the initial moment the mass of the substance is 50 mg. Six hours later the mass of the radioactive substance would be nearly_____.

Q.6 A point object is located at a distance of 30 cm on the principal axis of a convex lens of focal length 20 cm. If the lens is moved away from the object by 10 cm, find the distance through which the image moves.

Q.7 From amongst resistances of 2W, 3W, 6W and 7W, two resistors are picked and combined first in series and then in parallel. Their equivalent resistances are found to be 2W and 9W. Find the parallel equivalent of the remaining two resistances.

Q.8 Dust particles are uniformly distributed in space with a mass per unit volume ρ. A circular plate of area A is moving in this space at v. The dust particles that come into contact with the plate stick to it. The mass of dust that collects on the plate in a time t is _____.

Q.9 Three 60 W light bulbs are mistakenly wired in series and connected to a 120 V power supply. Assume the light bulbs are rated for single connection to 120 V. With the mistaken connection, the power dissipated by each bulb is_____.

Q.10 A candle flame and a screen are 180 cm apart. A lens is placed between them. For a certain position of the lens a twice magnified clear image is obtained on the screen. The lens is now moved away from the candle through 30 cm. To obtain another clear image on the screen, the distance through which the screen has to be moved is _____ and the new magnification of the image is _____.

CHEMISTRY

Q.1 An element of atomic weight X consists of two isotopes of Mass number (X – 1) and (X + 2). What is the percentage abundance of heavier isotope ?

Q.2 Bond angle in methane is 109°28′ since the carbon atom is sp^3 hybridised and all the H atoms form the vertices of a regular tetrahedral. We ask you to provide a convincing semi-mathematical reason based on geometry &/or energy arguments as to why the bond angle is 109°28′ & the methane molecule 3-dimensional instead of the bond angle being a right angle which may have been possible if the methane molecule was planar and the 4 H atoms were at the vertices of a square with C atom at the center.

Q.3 The molecules of SO_2 & H_20 have dipole moment but CO_2 has no dipole moment. Thus, we refer to SO_2, & H_20 as polar molecules and CO_2 as a non-polar molecule. The polarity of molecules is important in studying various properties like structure & reactivity, solvation, acidity-basicity etc. We ask you to suggest why CO_2 is a non-polar molecule. (Remember all the 3 molecules are covalently bonded)

Q.4 You have performed Acid-Base (or Acid-Alkali) Titration Reactions in your school laboratory. The general reaction is given as :

Acid + Base → Salt + Water

An indicator like phenolphthalein or methyl orange is used. It changes color at the time when neutralization is complete. The governing equation is :

$$N_1V_1 = N_2V_2$$

where, N is the Normality of the Acid / Base &
V is the volume in litres.

We ask you to give a scientific justification or derivation to the above equation i.e. Why must the product of Normality & Volume be equal for Acid & Base ?

Q.5 An experiment to measure Avogadro's number involves the collection of a beam of alpha particles to which electrons are added to give neutral helium. Suppose that for a particular run it takes a number of electrons equivalent to 6.40×10^{-5} coulomb/sec to neutralise the beam, and that after 24 hours of collection 1.14×10^{-4} g of neutral helium are collected. From this data, determine Avogadro's number. The charge on one electron is 1.602×10^{-19} coulombs. (24 hours is equivalent to 86,400 sec)